Two Type I Improper Integrals — One Convergent and One Not Convergent

A. Determine if $\int_2^\infty \frac{1}{(x-1)^2}\,dx$ is Convergent or NOT.

<u>Sol'n</u>: $\int_2^\infty \frac{1}{(x-1)^2}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{(x-1)^2}\,dx$

Use $u=x-1$ Substitution $du = dx$ When $x=2$, $u=1$. When $x=t$, $u=t-1$

$= \lim_{t\to\infty} \int_1^{t-1} \frac{1}{u^2}\,du$

$= \lim_{t\to\infty} \int_1^{t-1} u^{-2}\,du$ $\boxed{\frac{u^{-1}}{-1} = -\frac{1}{u}}$ $= \lim_{t\to\infty} \left(\left[-\frac{1}{u}\right]_1^{t-1} \right)$

$= \lim_{t\to\infty} \left(\left(-\frac{1}{t-1}\right) - \left(-\frac{1}{1}\right) \right) = \lim_{t\to\infty} \left(-\frac{1}{t-1} + 1 \right)$

$= 0 + 1 = 1$

So, $\int_2^\infty \frac{1}{(x-1)^2}\,dx$ is Convergent

and $\int_2^\infty \frac{1}{(x-1)^2}\,dx = 1$.

B. Determine if $\int_2^\infty \frac{1}{x-1}\,dx$ is Convergent or NOT.

Sol'n $\int_2^\infty \frac{1}{(x-1)}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{(x-1)}\,dx$

Use $u = x-1$ Substitution
$du = dx$
When $x=2$, $u=1$
When $x=t$, $u=t-1$

$= \lim_{t\to\infty} \int_1^{t-1} \frac{1}{u}\,du$

$= \lim_{t\to\infty} \left(\left[\ln u\right]_1^{t-1} \right)$

$= \lim_{t\to\infty} \left(\ln(t-1) - \ln(1) \right) = \lim_{t\to\infty} \left(\ln(t-1) \right)$

[Since $\ln(1) = 0$]

$= \infty$, since $t-1 \to \infty$ as $t \to \infty$.

$\therefore \int_2^\infty \frac{1}{x-1}\,dx = \infty$, so

$\int_2^\infty \frac{1}{x-1}\,dx$ is Divergent.